

An Introduction to Chaos Theory Workshop

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Concepts of Order and Disorder

- *What is Order ?!!!*
- *... and Disorder*
- *Why Order ?!!!*
- *... and Disorder*
- *Hybrid Systems*

Causality, Determinism, and Randomness

- *Causality*
- *Determinism*
- *Freewill*
- *Randomness*
- *Goal Directed Action*

Intelligence and Complexity

- *What is Intelligence?*
- *Artificial Intelligence ?!!!*
- *Simple Behaviors*
- *Complex Behaviors*
- *Conclusion*

Introduction to State-Space

- *System Modeling*
- *Dynamical Systems*

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) \quad , \quad \mathbf{x}(t_0) = \mathbf{x}_0 \quad , \quad \mathbf{x}(t) \in \mathcal{R}^m$$

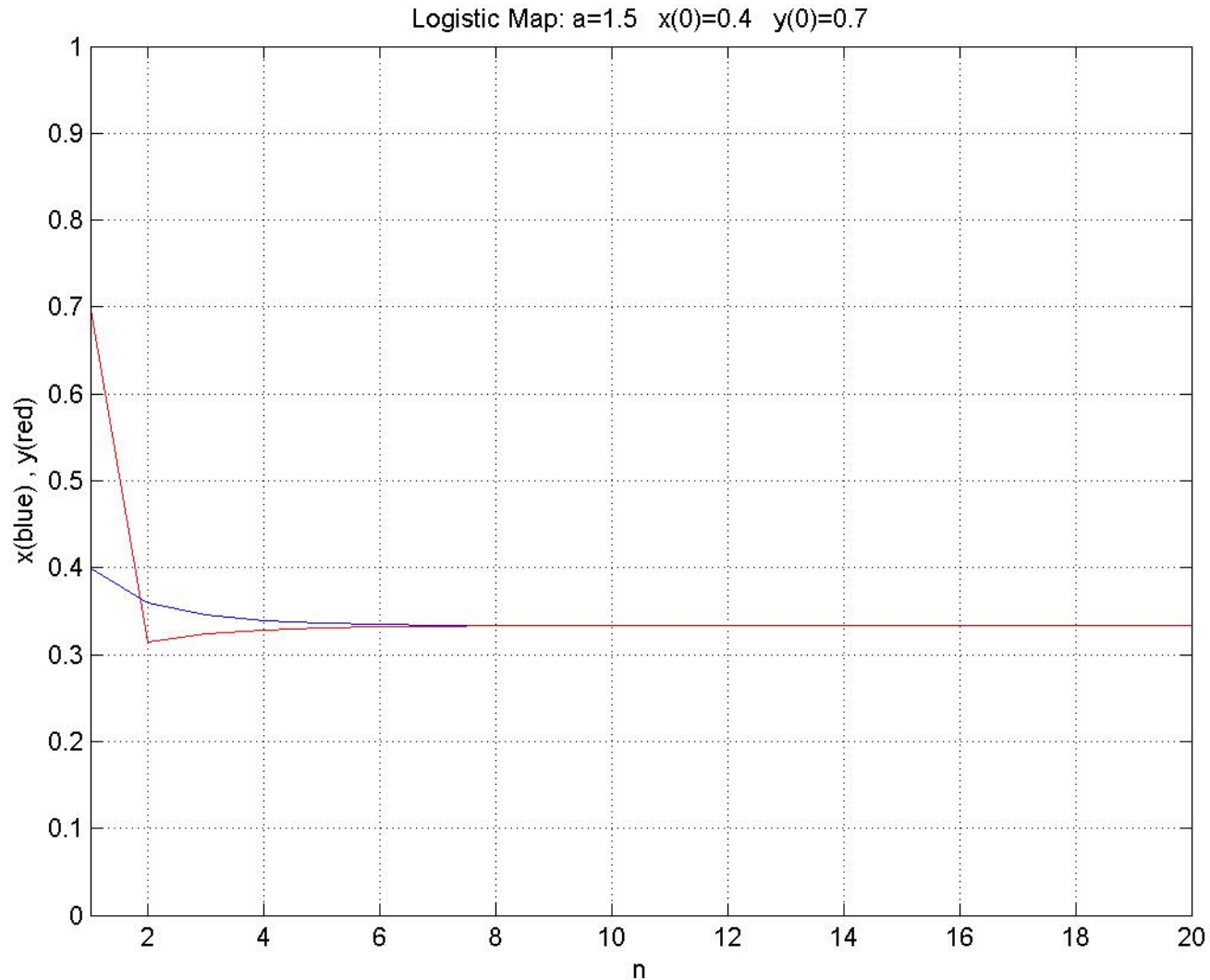
$$\mathbf{x}_{n+1} = \mathbf{f}(\mathbf{x}_n) \quad , \quad \mathbf{x}_0 = \mathbf{x}_0 \quad , \quad \mathbf{x}_n \in \mathcal{R}^m$$

- *Example: Logistic Map*

$$\mathbf{x}_{n+1} = a\mathbf{x}_n(1 - \mathbf{x}_n)$$

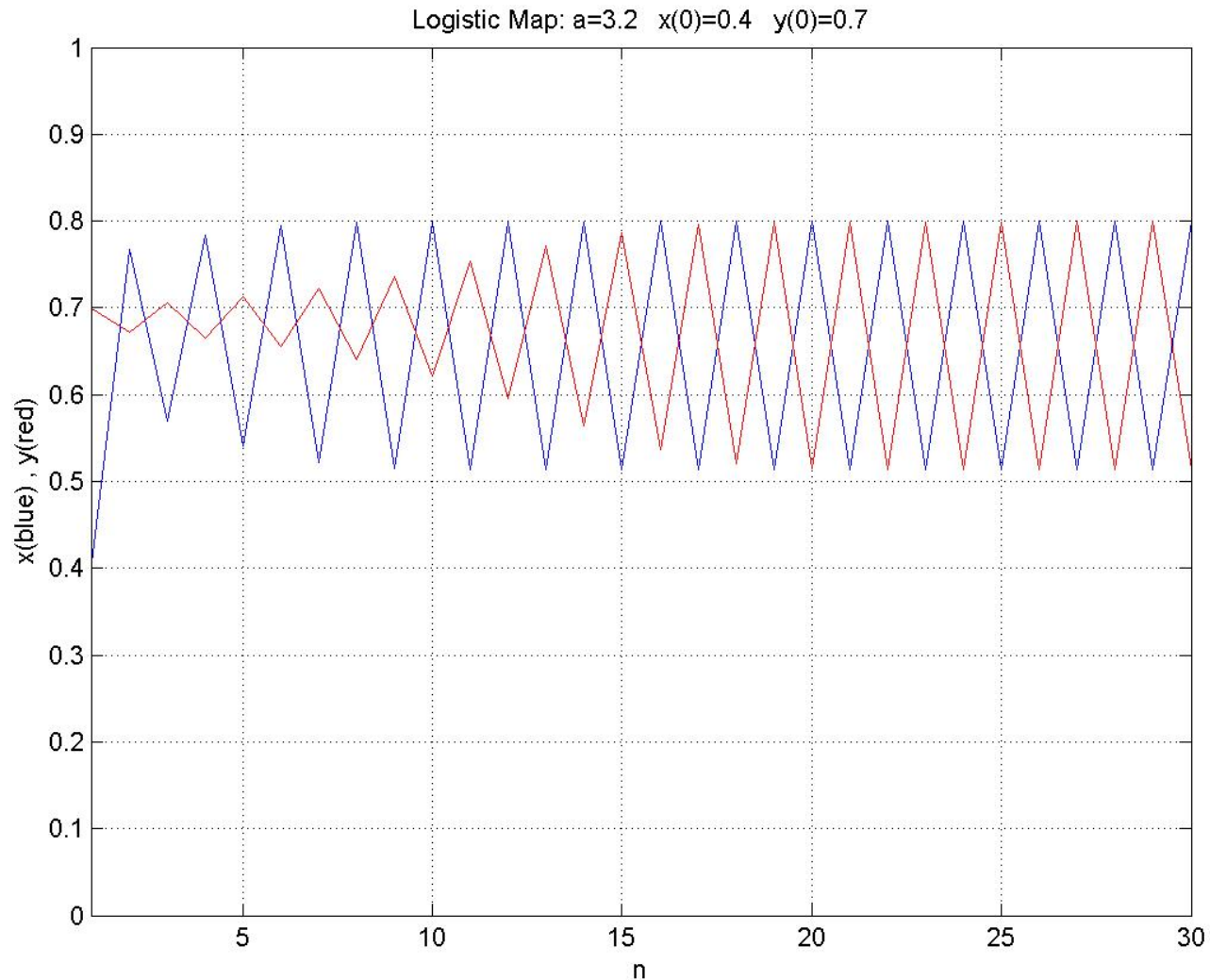
Steady-State Behavior – I

Fixed Point



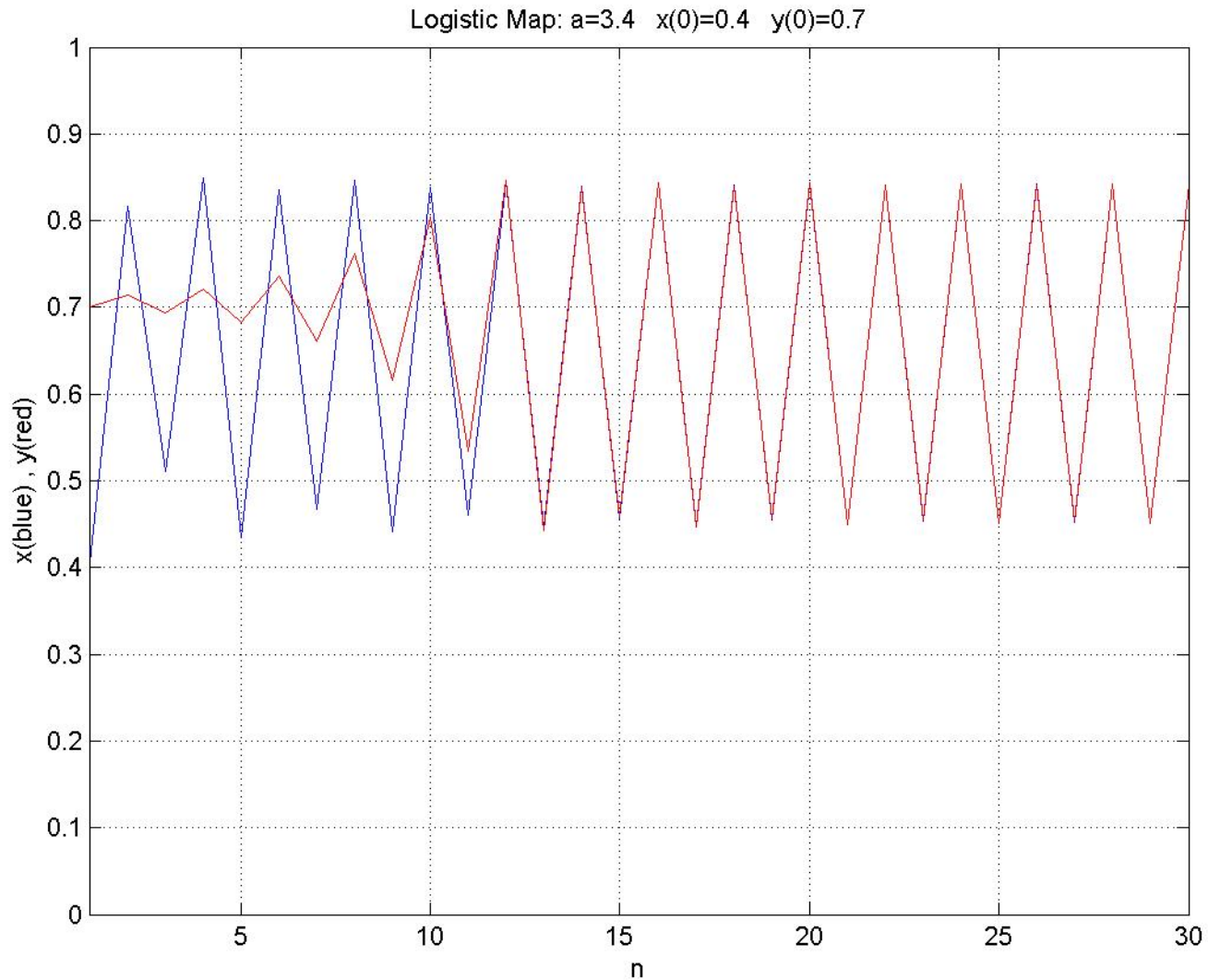
Steady-State Behavior – II

Limit Cycle



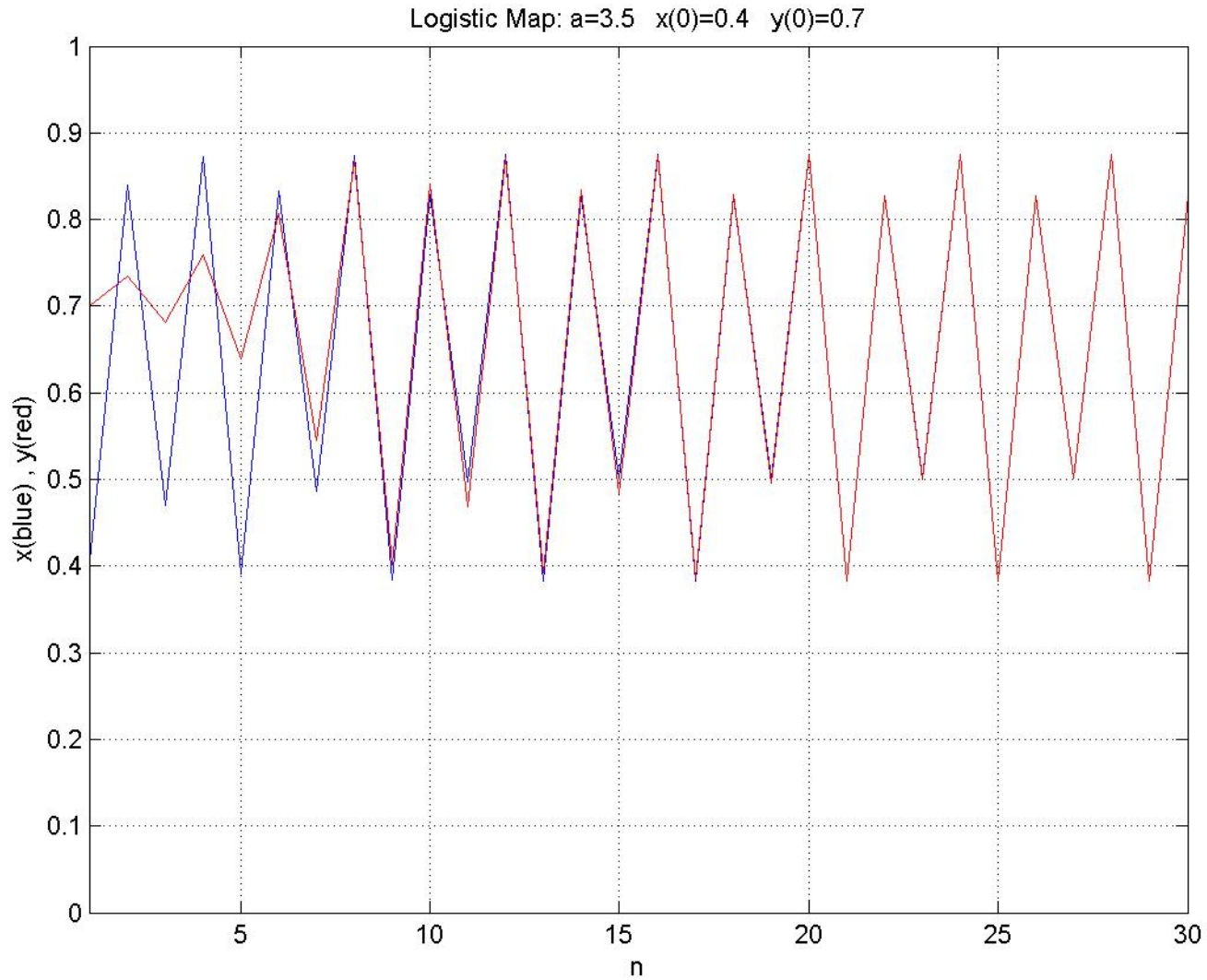
Steady-State Behavior – II

... Limit Cycle



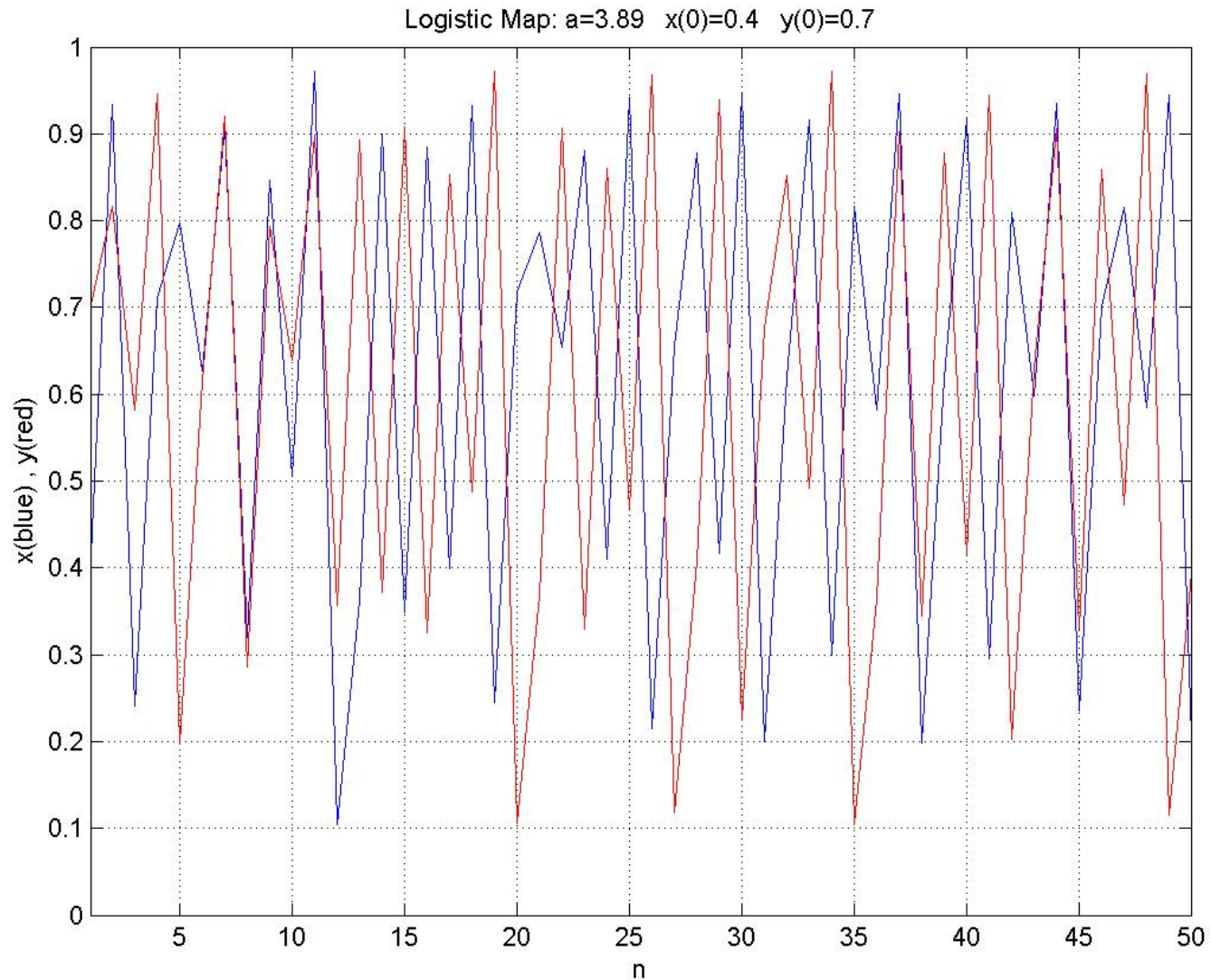
Steady-State Behavior - IV

Quasi-periodic



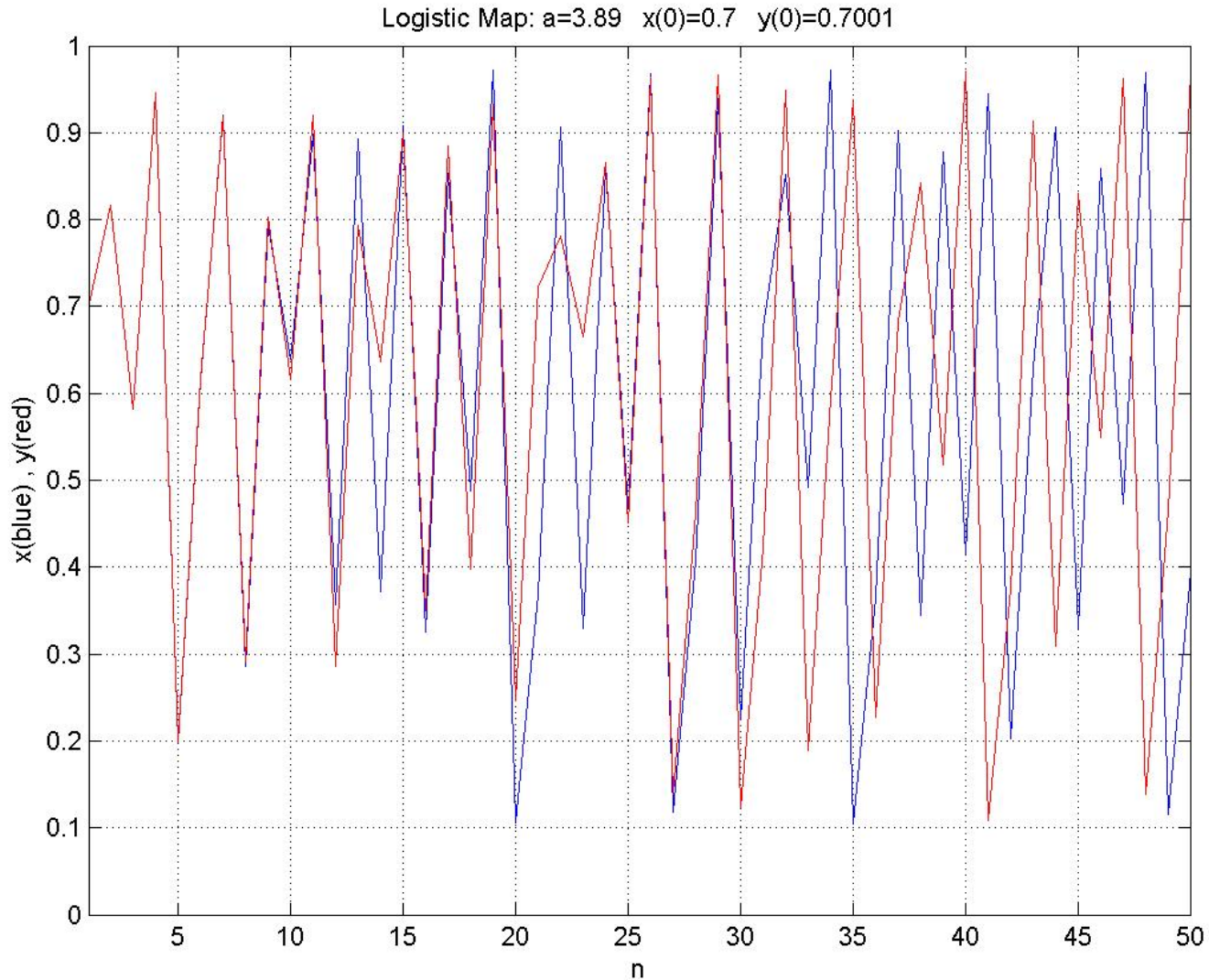
Steady-State Behavior - V

Chaos



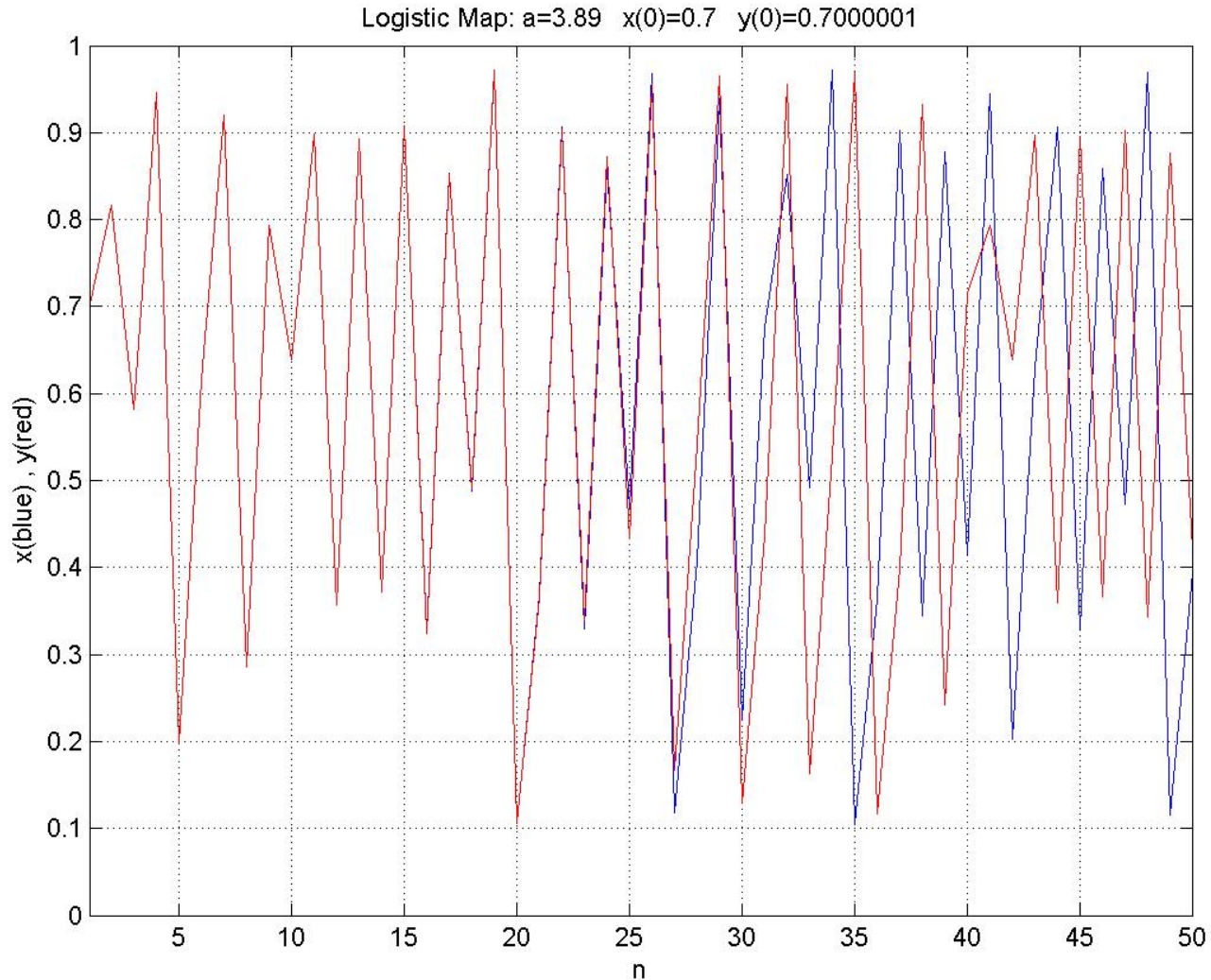
Steady-State Behavior - V

... Chaos



Steady-State Behavior - V

... Chaos



Definition of Chaos - I

● *Historical Perspective*

- Laplace , 1776
- Poincare, 1903
- Birkhoff, 1920
- Kolmogorov, 1960
- Lorenz, 1963
- ...

Definition of Chaos - II

- *None of the rest !!!*
- *Deterministic Randomness*
- *Unpredictable Causality*

Chaotic Systems Properties - I

- *Non-periodic Bounded Motion with Initial Condition Sensitivity*
 - *Butterfly Effect !!!*
 - *Restless Motion !!!*
 - *Heisenberg's Uncertainty Principle*
 - *Naturally Unpredictable !!!*
- *Continuous, Broad-band, Noise-like Power Spectrum*

Chaotic Systems Properties - II

● *Stretching and Folding*

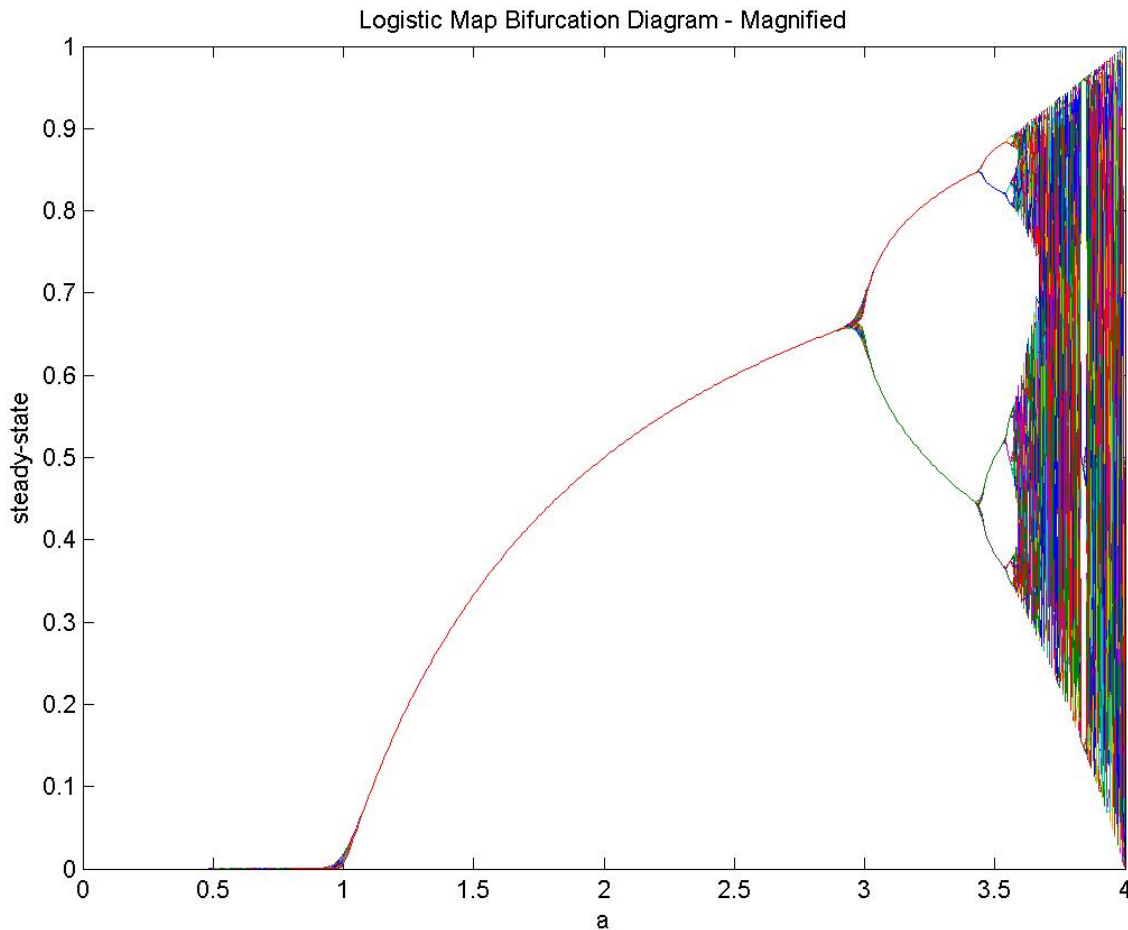
- *Lyapunov Exponents*
- *Growing Uncertainty*

● *Information Creation and Destruction*

- *2nd Law of Thermodynamics Suspected*

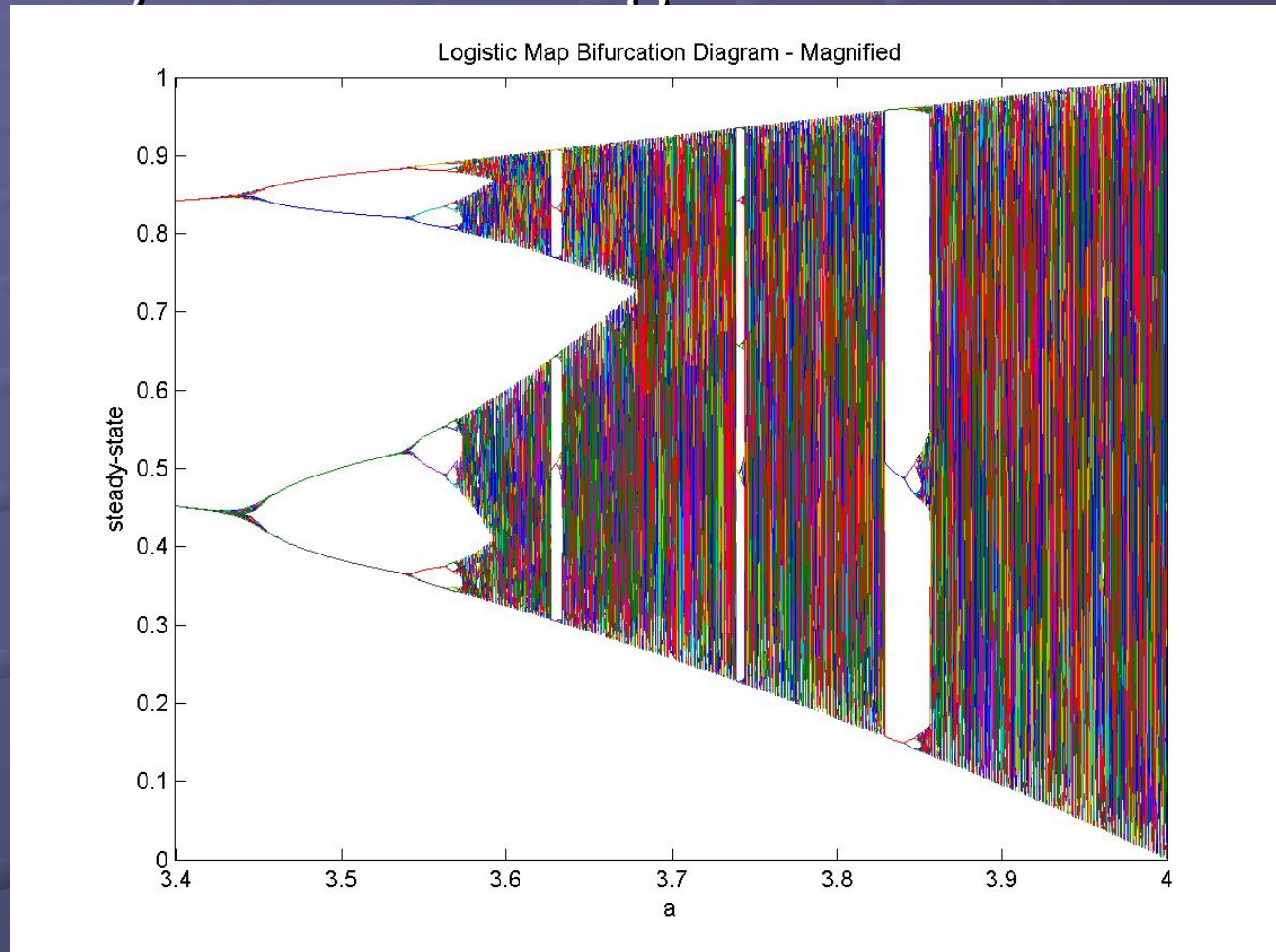
Chaotic Systems Properties - III

Bifurcation Diagram



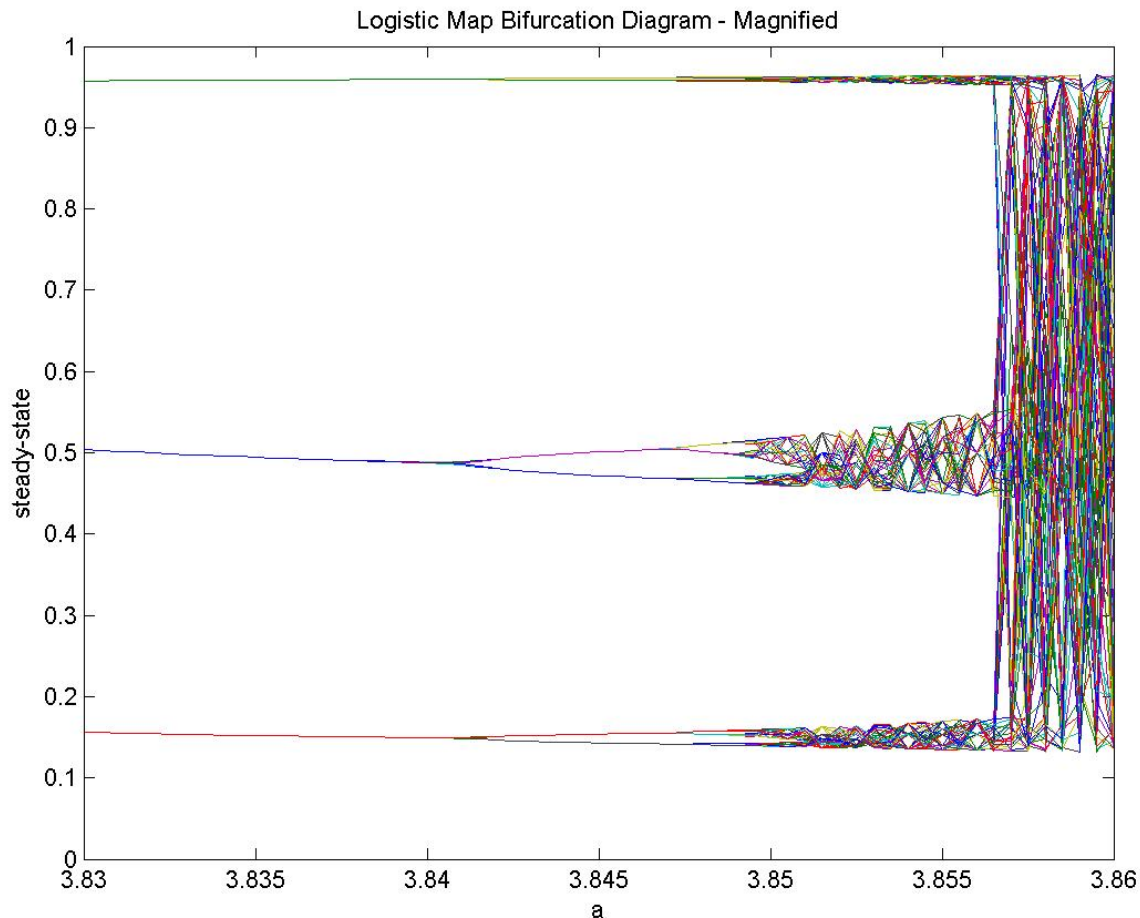
Chaotic Systems Properties - III

... *Bifurcation Diagram*



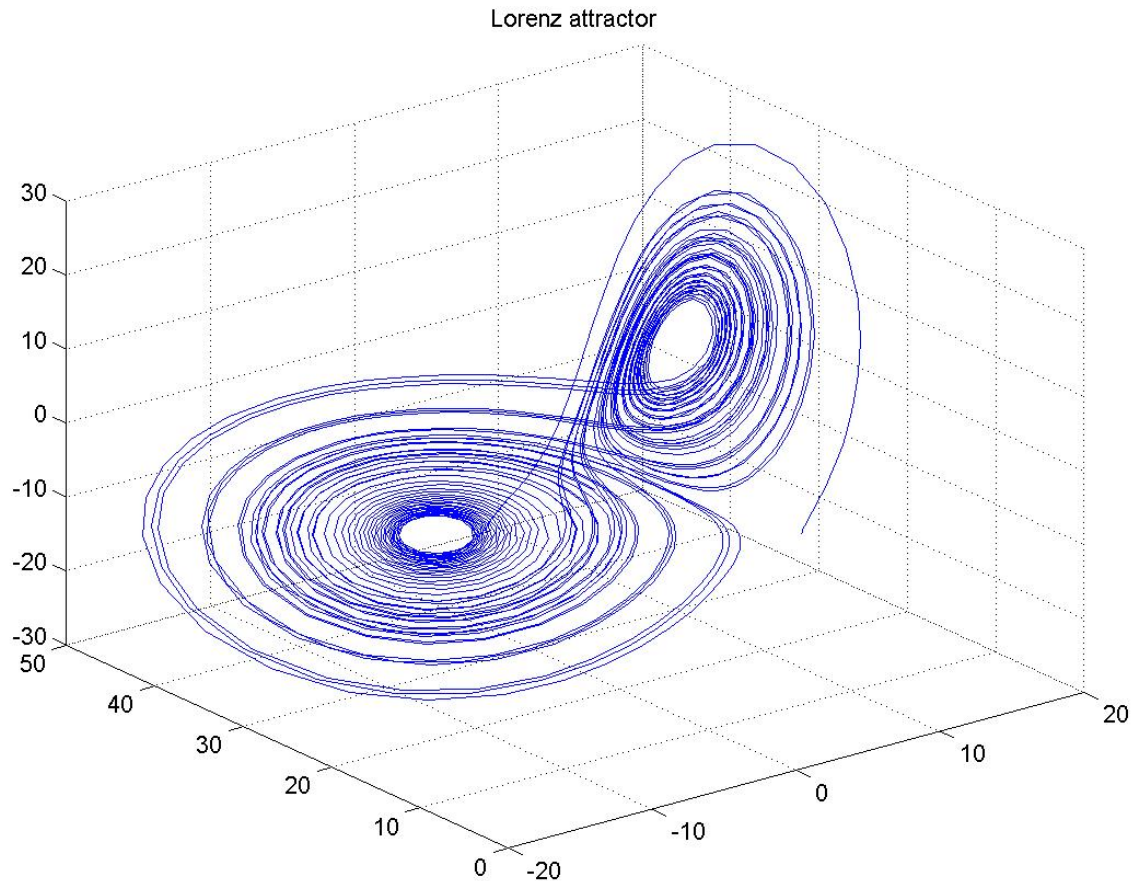
Chaotic Systems Properties - III

● ... *Bifurcation Diagram*



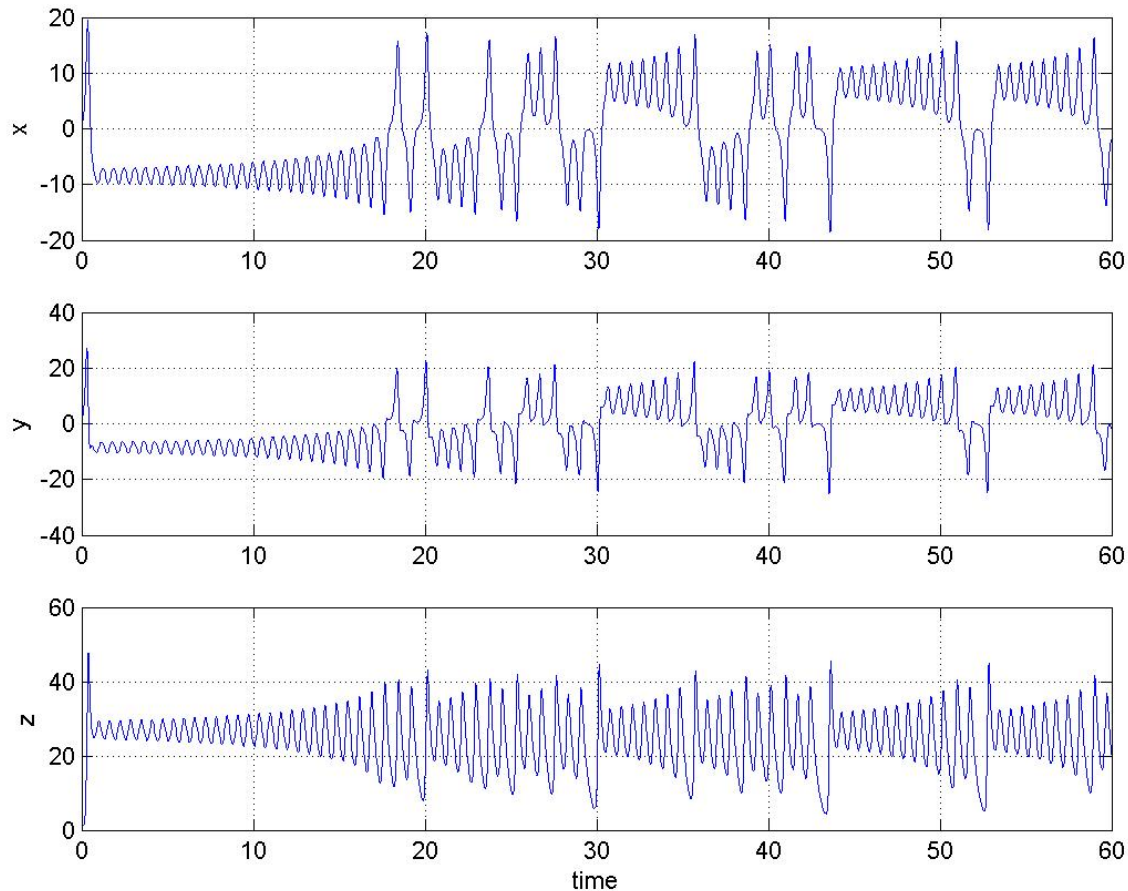
Chaotic Systems Properties - IV

● *Strange Attractor*



Chaotic Systems Properties - IV

● ... *Strange Attractor*

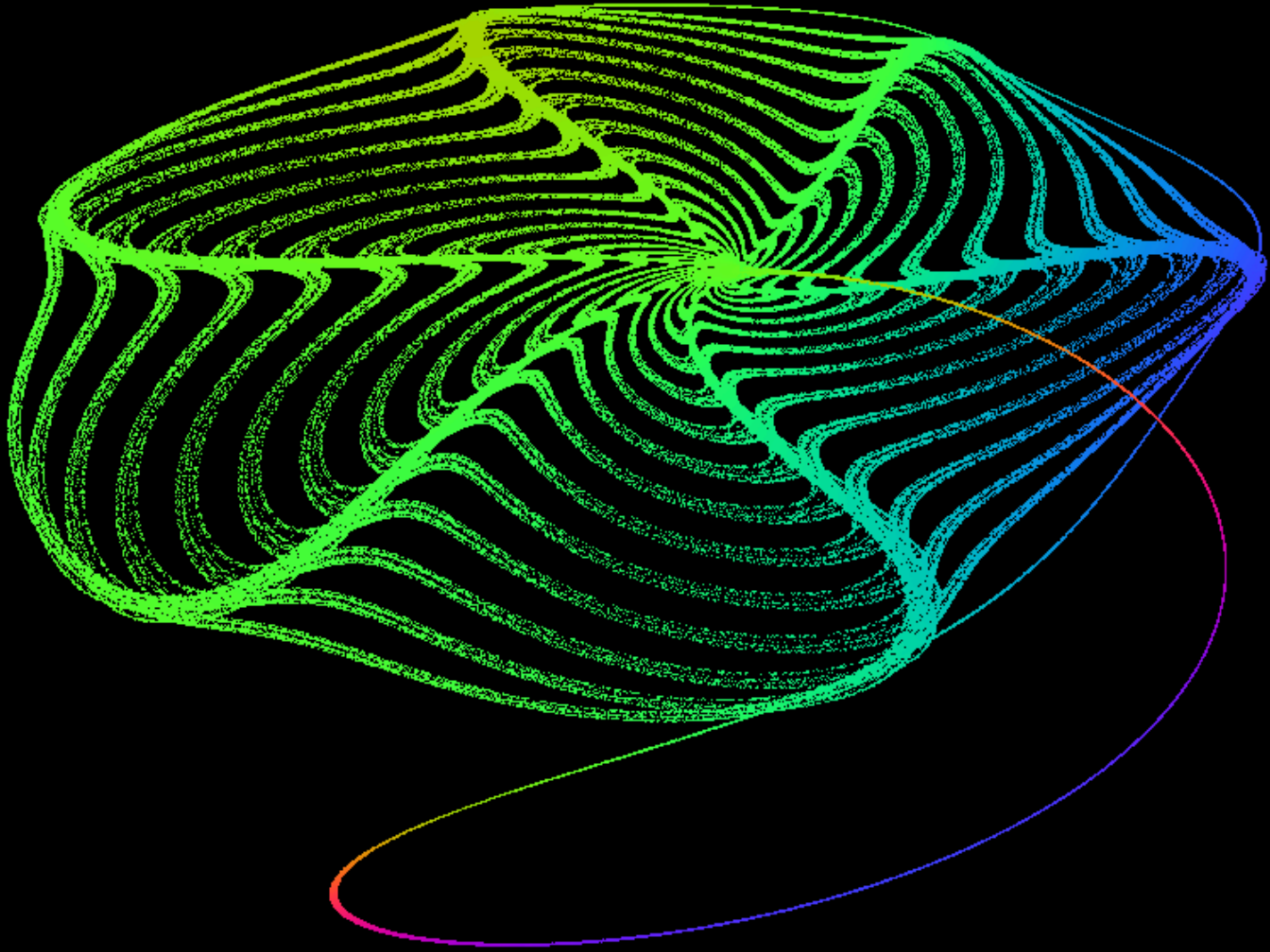


More Strange Attractors

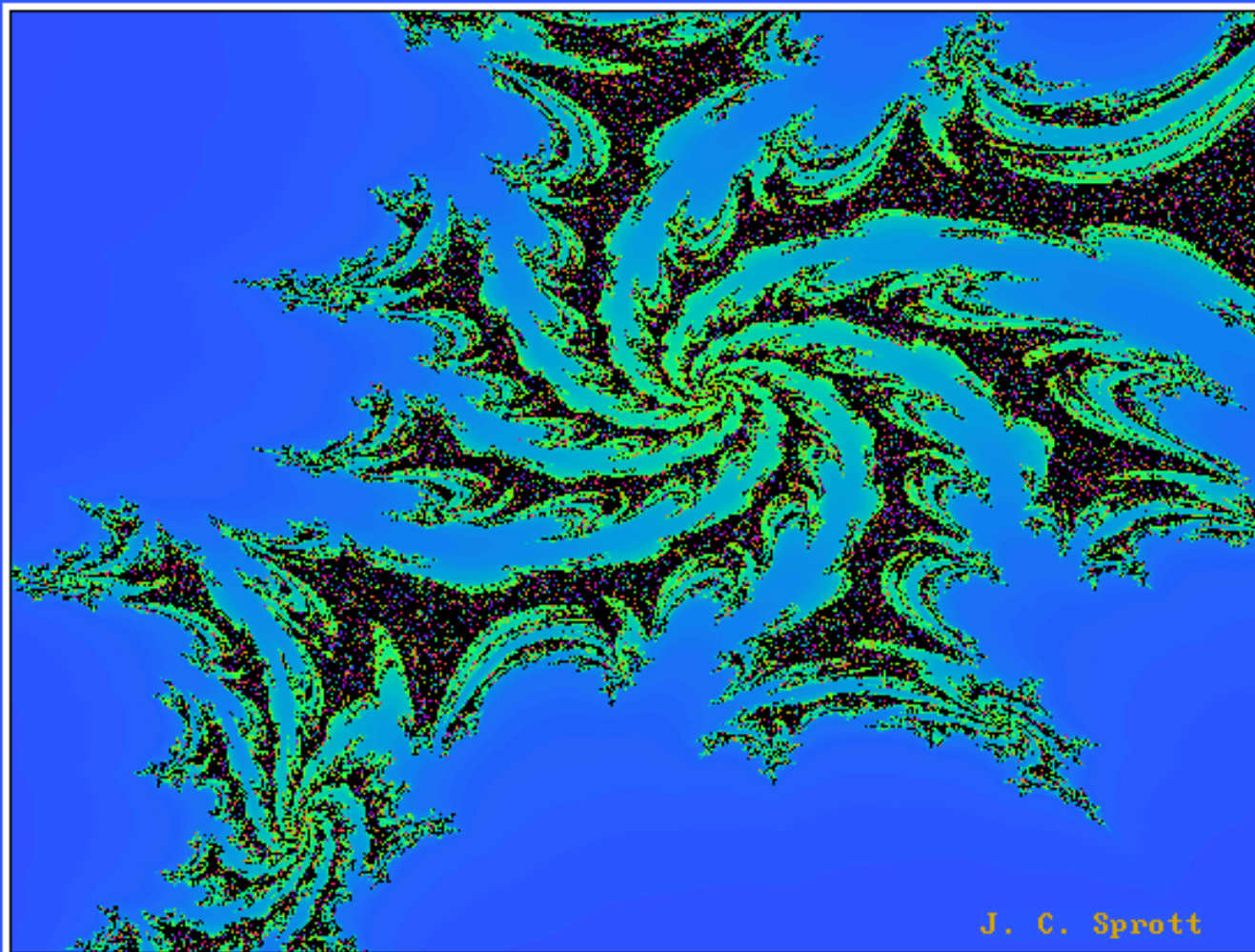
Strange Attractor

INRRXLCEYLFHWAPFSTPHHJMYRYJFBNM

J. C. Sprott



More Strange Attractors



J. C. Sprott

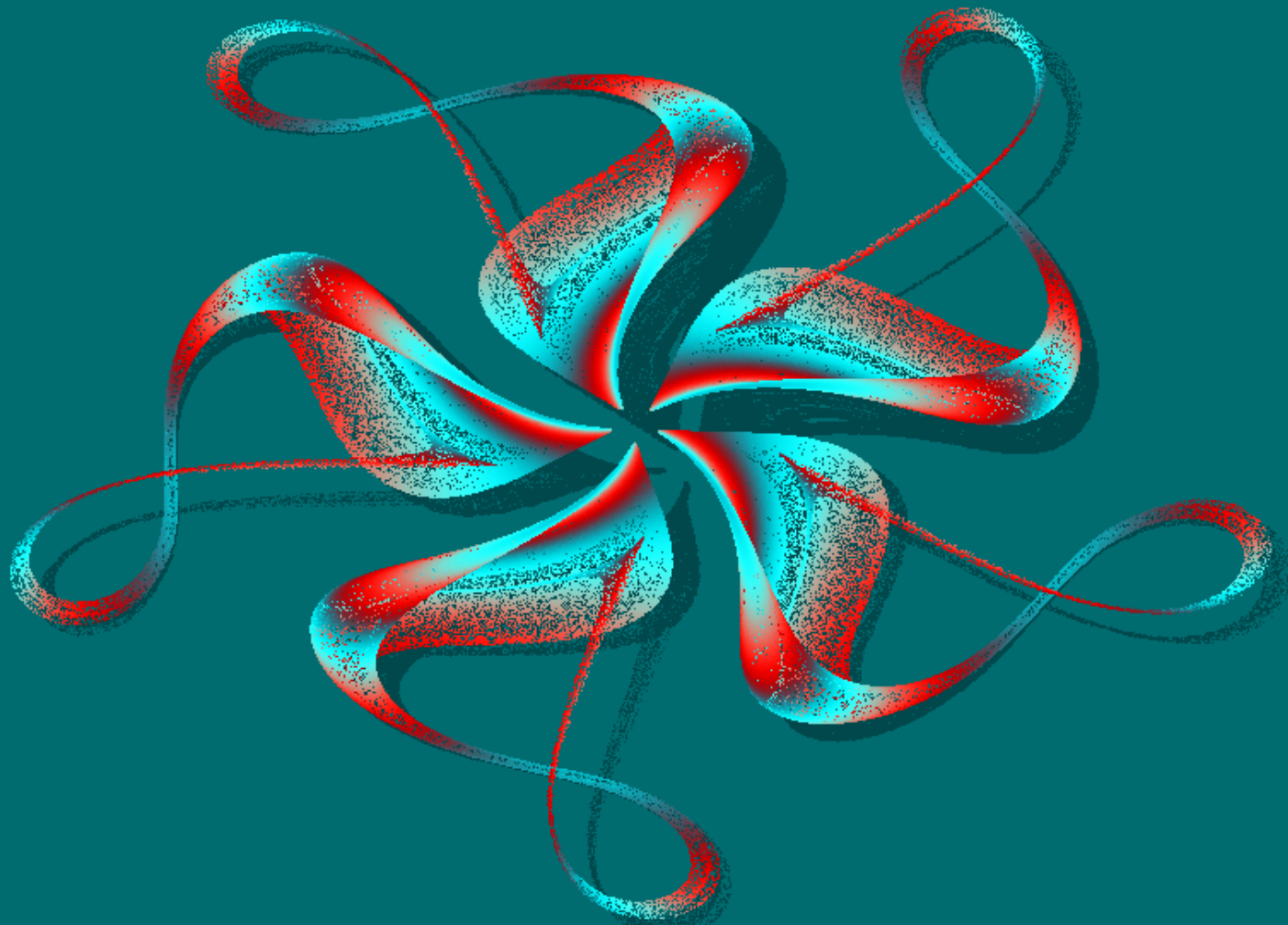
Quadratic Basins
EPARXHHPRGSAS

More Strange Attractors

Strange Attractor icon

QOFNACAWLADXBPARLHLYGEYVKXCTUJP

J. C. Sprott

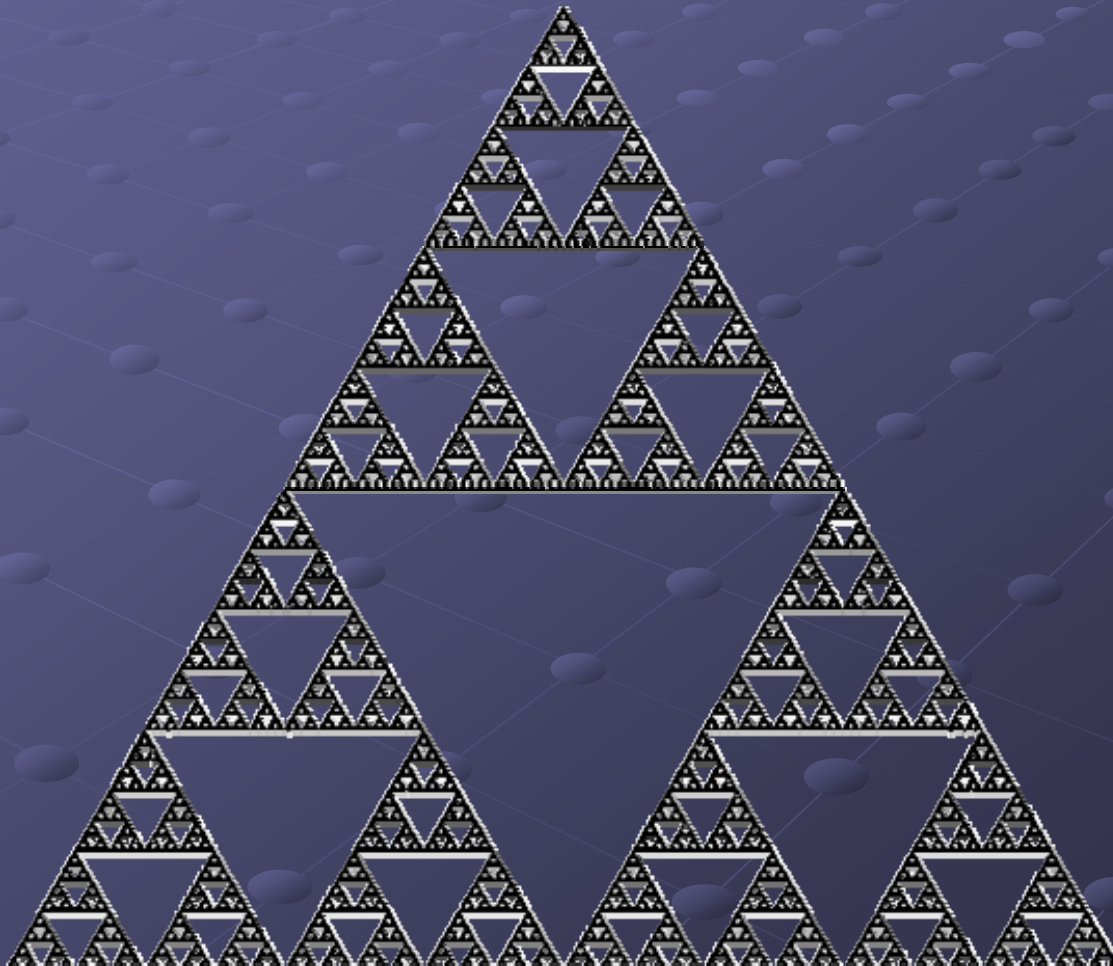


Fractal Dimension

- *What is Dimension?*
- *Why Euclidian?*
- *What is Fractal?*
- *What is Fractal Dimension*

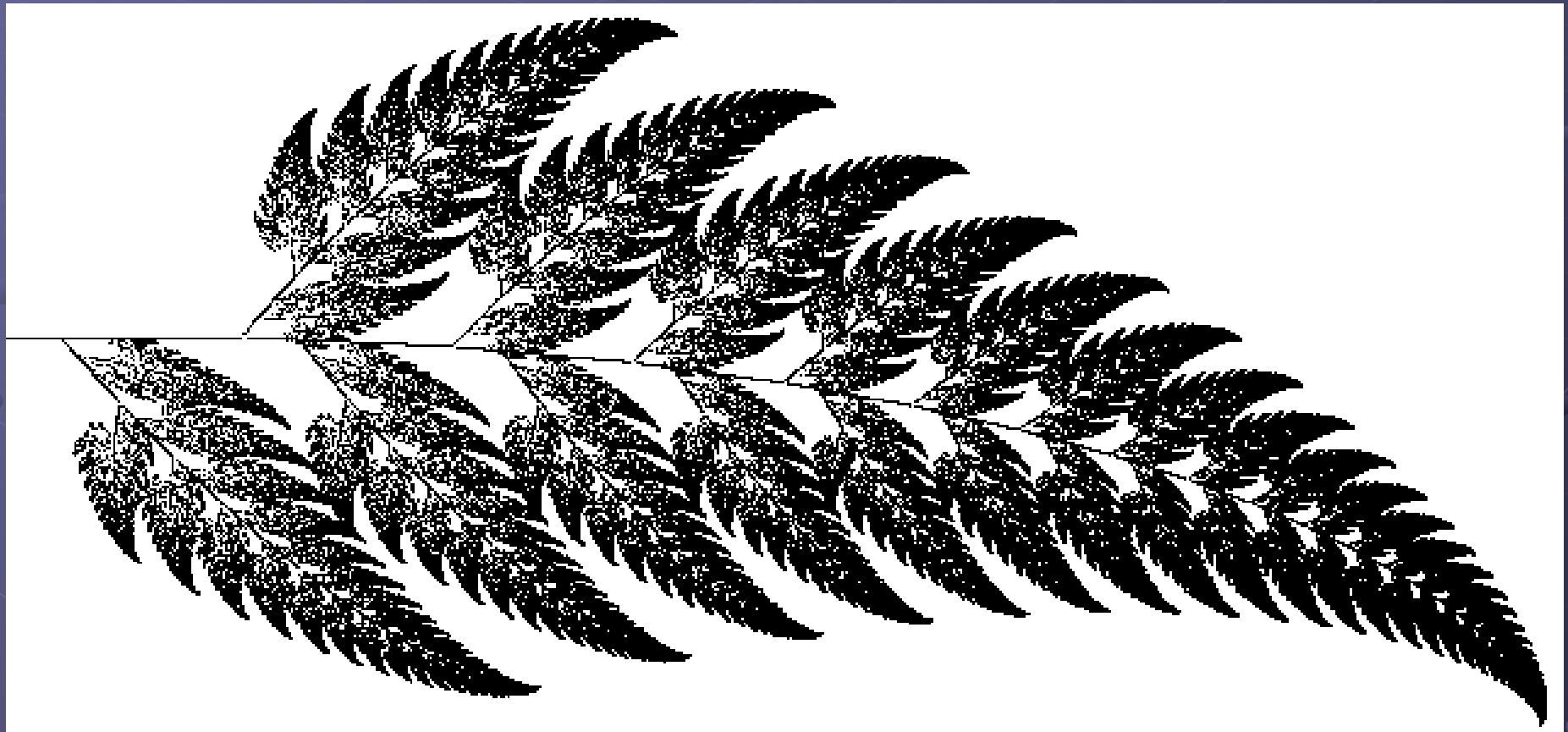
More Fractals

● *Sierpinsky Triangle*



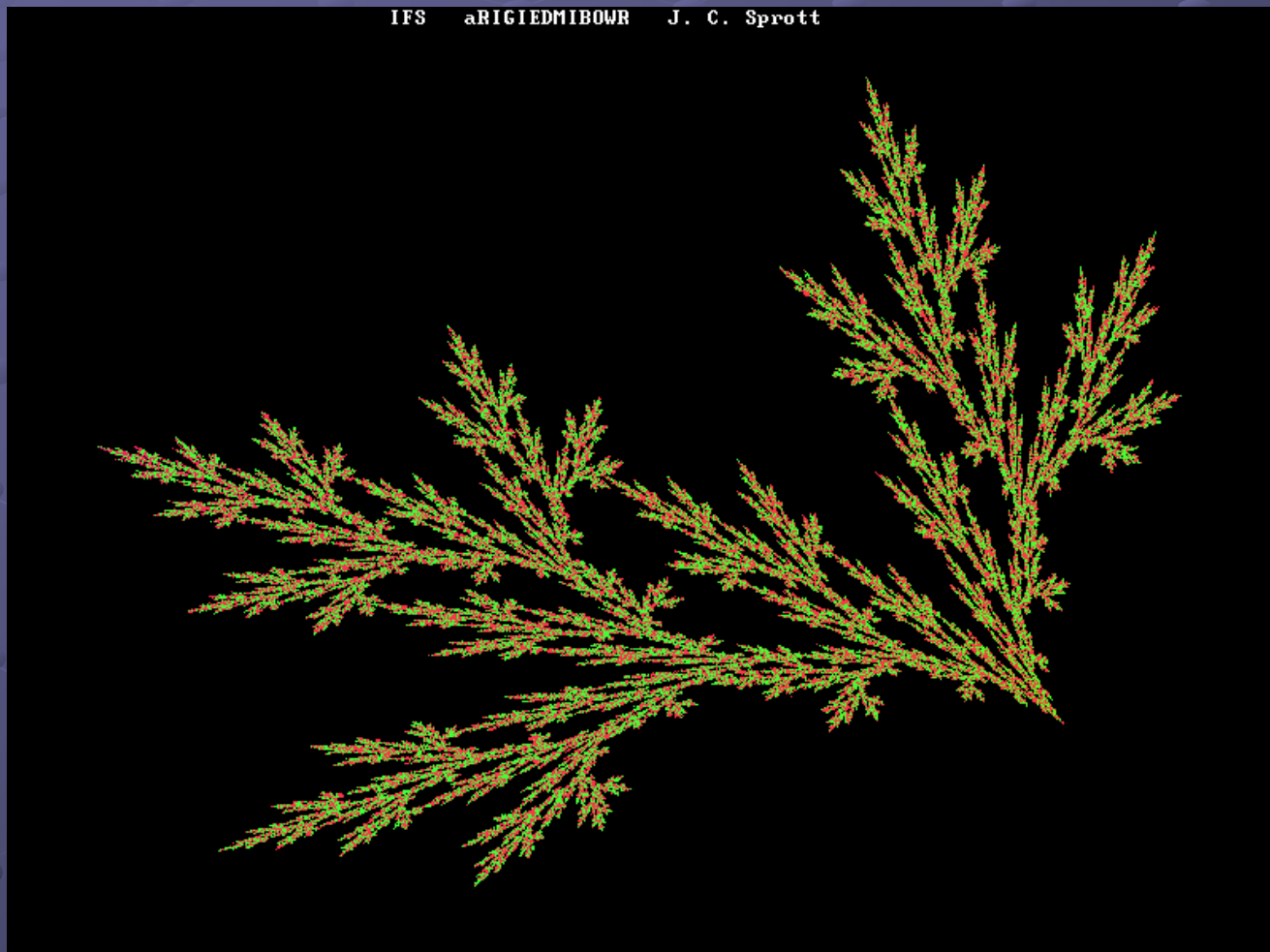
More Fractals

● *Fern*



More Fractals

● *Even Completer*



More Fractals

● *And Natural Ones*

