Adaptive Stabilization of Lorenz Chaos

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What's the Chaos?

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Invariant Manifold

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Adaptive Control of Lorenz Chaos with Unknown Parameters

Lorenz System:

$$\dot{x}_1 = \sigma(x_2 - x_1), \quad (1)$$
$$\dot{x}_2 = \rho x_1 - x_1 x_3 - x_2, \quad (2)$$
$$\dot{x}_3 = -\beta x_3 + x_1 x_2. \quad (3)$$

Unstable fixed points

$$a = (a_1, a_2, a_3)^{\mathrm{T}} = \begin{bmatrix} (\beta(\rho-1))^{1/2} \\ (\beta(\rho-1))^{1/2} \\ \rho - 1 \end{bmatrix}, \quad (4)$$
$$b = (b_1, b_2, b_3)^{\mathrm{T}} = \begin{bmatrix} -(\beta(\rho-1))^{1/2} \\ -(\beta(\rho-1))^{1/2} \\ \rho - 1 \end{bmatrix}, \quad (5)$$
$$c = (c_1, c_2, c_3)^{\mathrm{T}} = (0, 0, 0)^{\mathrm{T}}. \quad (6)$$

Modified Lorenz System with Control Inputs:

$$\dot{x}_1 = \sigma(x_2 - x_1), \quad (7)$$
$$\dot{x}_2 = \rho x_1 - x_2 - x_1 x_3 + u_1, \quad (8)$$
$$\dot{x}_3 = -\beta x_3 + x_1 x_2 + u_2. \quad (9)$$

Selecting Invariant Manifolds, Stable and Unstable:

Stability at Intersection of Manifolds:

$$M_c = M_1 \cap M_2 = \{x \in R^3 : x_2 - a_2 = 0, x_3 - a_3 = 0\}. \qquad (12)$$

Lasalle's Invariant Set Principle

Controller Designing with Known Parameters:

$$V(x) = \frac{1}{2}\psi^T(x)T\psi(x) \qquad (13)$$

$$T\dot{\psi}(x) + \psi(x) = 0. \qquad (14)$$

$$\dot{V}(x) = -\psi^T(x)\psi(x). , \quad \dot{V}(x) < 0$$

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = -\begin{bmatrix} \rho x_1 - x_1 x_3 - x_2 \\ -\beta x_3 + x_1 x_2 \end{bmatrix} - T^{-1}\begin{bmatrix} x_2 - a_2 \\ x_3 - a_3 \end{bmatrix}. \qquad (15)$$

Controller Designing with Unknown Parameters:

$$\hat{\psi}_1(x) = x_2 - \hat{a}_2 \text{ and } \hat{\psi}_2(x) = x_3 - \hat{a}_3, \quad \hat{a}_2 = \sqrt{\hat{\beta}(\hat{\rho}-1)}, \quad \hat{a}_3 = \hat{\rho} - 1$$

$$V(x, \hat{\rho}, \hat{\beta}) = \frac{1}{2}\hat{\psi}^T(x)T\hat{\psi}(x) + e^T\Gamma^{-1}e. \qquad (16)$$

$$e = \theta - \hat{\theta}, \quad \theta = [\ \rho \quad \beta\ ]^T, \quad \hat{\theta} = [\ \hat{\rho} \quad \hat{\beta}\ ]^T, \quad \hat{\psi}^T(x) = [\ \hat{\psi}_1 \quad \hat{\psi}_2\ ]$$

$$f(x) = \begin{bmatrix} \sigma(x_2 - x_1) \\ -x_2 - x_1 x_2 \\ x_1 x_2 \end{bmatrix}, \quad F(x) = \begin{bmatrix} 0 & 0 \\ x_1 & 0 \\ 0 & -x_3 \end{bmatrix}, \quad g(x) = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

$$\begin{aligned} \dot{V}(x, \hat{\rho}, \hat{\beta}) &= \hat{\psi}^T T \frac{\partial \hat{\psi}}{\partial x}(f(x) + F(x)\theta + g(x)u) + \hat{\psi}^T T \frac{\partial \hat{\psi}}{\partial \hat{\theta}}\dot{\hat{\theta}} - e^T\Gamma^{-1}\dot{\hat{\theta}} \\ &= \hat{\psi}^T T \frac{\partial \hat{\psi}}{\partial x}(f(x) + F(x)\hat{\theta} + g(x)u) + \hat{\psi}^T T \frac{\partial \hat{\psi}}{\partial x}F(x)e + \hat{\psi}^T T \frac{\partial \hat{\psi}}{\partial \hat{\theta}}\dot{\hat{\theta}} - e^T\Gamma^{-1}\dot{\hat{\theta}}. \end{aligned}$$

$$u = u_e + u_c.$$

$$\frac{\partial\hat{\psi}}{\partial x}(f(x) + F(x)\hat{\theta} + g(x)u_e) = -T^{-1}\hat{\psi}, \tag{17}$$

$$\hat{\psi}^T T \frac{\partial\hat{\psi}}{\partial x} g(x) u_c = -\hat{\psi}^T T \frac{\partial\hat{\psi}}{\partial\hat{\theta}}\dot{\hat{\theta}}, \tag{18}$$

$$\hat{\psi}^T T \frac{\partial\hat{\psi}}{\partial x} F(x) e = e^T \Gamma^{-1}\dot{\hat{\theta}}. \tag{19}$$

$$\dot{V} = -\hat{\psi}^T\hat{\psi} < 0$$

Control Inputs:

$$u_e = \begin{bmatrix} u_{e1} \\ u_{e2} \end{bmatrix} = -\begin{bmatrix} \hat{\rho}x_1 - x_1x_3 - x_2 \\ -\hat{\beta}x_3 + x_1x_2 \end{bmatrix} - T^{-1}\begin{bmatrix} x_2 - \hat{a}_2 \\ x_3 - \hat{a}_3 \end{bmatrix}, \tag{20}$$

$$u_c = \begin{bmatrix} u_{c1} \\ u_{c2} \end{bmatrix} = -\begin{bmatrix} -\frac{\hat{\beta}}{2\sqrt{\hat{\beta}(\hat{\rho}-1)}} & -\frac{(\hat{\rho}-1)}{2\sqrt{\hat{\beta}(\hat{\rho}-1)}} \\ -1 & 0 \end{bmatrix} \Gamma \begin{bmatrix} x_1 & 0 \\ 0 & -x_3 \end{bmatrix} T \begin{bmatrix} x_2 - \hat{a}_2 \\ x_3 - \hat{a}_3 \end{bmatrix}, \tag{21}$$

Adaptation Law:

$$\dot{\hat{\theta}} = \begin{bmatrix} \dot{\hat{\rho}} \\ \dot{\hat{\beta}} \end{bmatrix} = \Gamma\left[\frac{\partial\hat{\psi}}{\partial x}F(x)\right]^T T\hat{\psi} = \Gamma\begin{bmatrix} x_1 & 0 \\ 0 & -x_3 \end{bmatrix} T \begin{bmatrix} x_2 - \hat{a}_2 \\ x_3 - \hat{a}_3 \end{bmatrix}. \tag{22}$$

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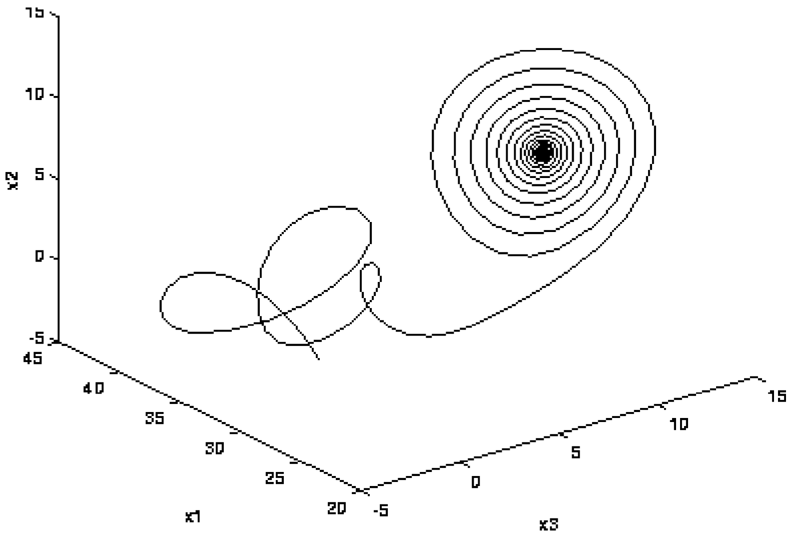
Simulation Results

Simulation Parameters:

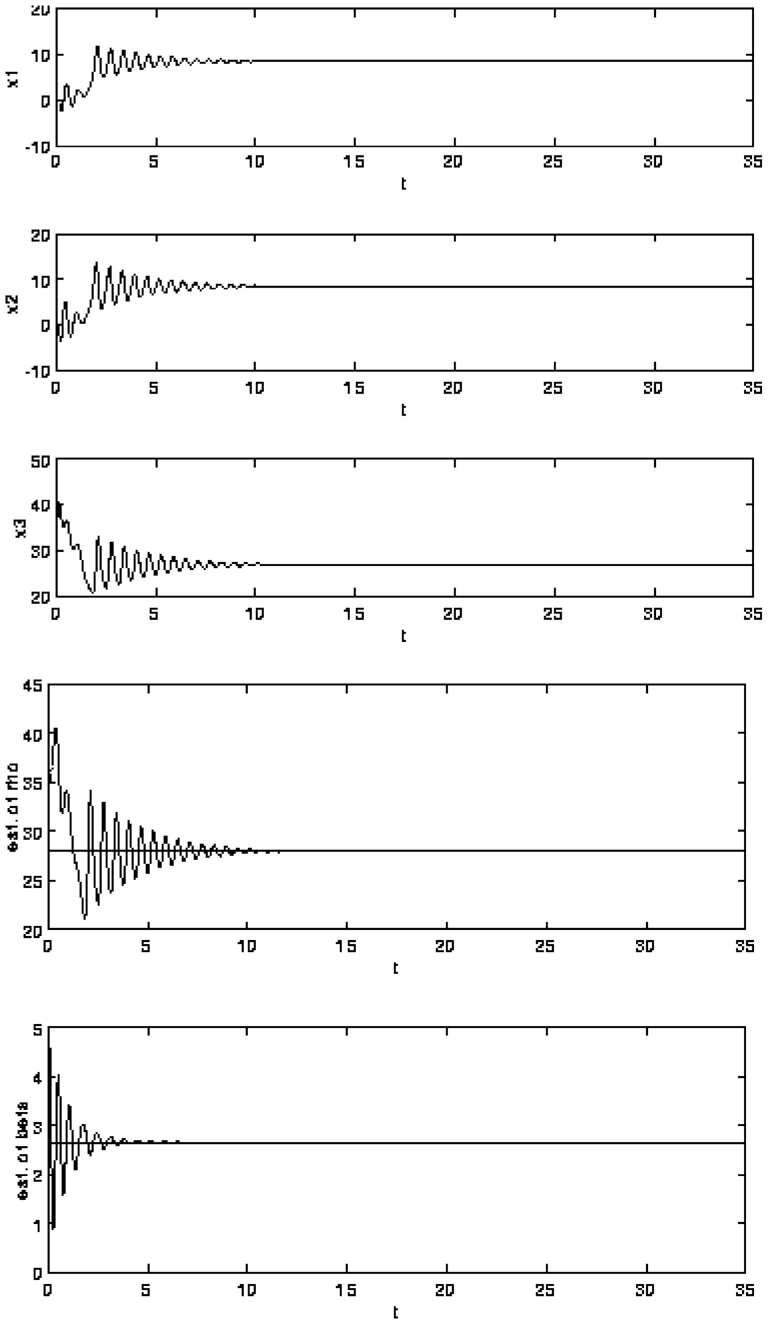
$\sigma = 10$, $\rho = 28$, $\beta = 8/3$

$$T = \begin{bmatrix} 1.43 & 0 \\ 0 & 0.50 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 1.05 & 0 \\ 0 & 0.25 \end{bmatrix},$$

$x(0) = (\ -2.0 \quad -1.0 \quad 30.0\ )^T$, $(\ \hat{\rho}_0 \quad \hat{\beta}_0\ ) = (\ 33.0 \quad 4.0\ )$



Simulation Results



My Simulations:

1) System with Fixed Unknown Parameters

2) System with Fixed Unknown

Parameters and Noisy State Feedback

3) System with Fixed Unknown and Time-Varing Parameters

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Discussion

Why We Need an Unstable Manifold for Stabilization?

$$\dot{x}_1 = \sigma(x_2 - x_1), \tag{26}$$

$$\dot{x}_2 = \rho x_1 - x_2 - x_1 x_3 + u, \tag{27}$$

$$\dot{x}_3 = -\beta x_3 + x_1 x_2. \tag{28}$$

$$V = \frac{1}{2}(x_2 - \hat{a}_2)^2 + \frac{1}{2}(\rho - \hat{\rho})^2 + \frac{1}{2}(\beta - \hat{\beta})^2. \tag{29}$$

$$u_e = -(\hat{\rho} x_1 - x_1 x_3 - \hat{a}_2), \tag{30}$$

$$u_c = \frac{x_1 \hat{\beta}(x_2 - \hat{a}_2)}{2\sqrt{\hat{\beta}(\hat{\rho} - 1)}}, \tag{31}$$

$$\begin{bmatrix} \dot{\hat{\rho}} \\ \dot{\hat{\beta}} \end{bmatrix} = \begin{bmatrix} (x_2 - \hat{a}_2)x_1 \\ 0 \end{bmatrix}. \tag{32}$$

$$x_e' = \begin{bmatrix} \sqrt{\hat{\beta}(\rho - 1)} \\ \sqrt{\hat{\beta}(\rho - 1)} \\ \frac{\hat{\beta}(\rho - 1)}{\beta} \end{bmatrix}.$$

Notes:

1) When the equilibrium to be stabilized is a function of unknown parameters, the system stability is dependent on the convergence of parameter estimation.

2) To ensure the convergence of parameter estimation we must introduce enough invariant manifolds to excite the system dynamics.

Ref.:

[1] Y. Tian, X. Yu, "Adaptive Stabilization of Lorenz Chaos", Complexity International, Vol.6, 1998.

[2] E. Ott, "Chaos in Dynamical Systems", Cambridge Universuty Press, 1993.