An Introduction to Chaos Theory Workshop

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Philosophical Aspects
- Concepts of Order and Disorder
- Causality, Determinism, and Randomness
- Intelligence and Complexity

Mathematical Aspects
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Concepts of Order and Disorder

- What is Order ?!!!
- ... and Disorder
- Why Order ?!!!
- ... and Disorder
- Hybrid Systems
Causality, Determinism, and Randomness

- Causality
- Determinism
- Freewill
- Randomness
- Goal Directed Action
Intelligence and Complexity

- What is Intelligence?
- Artificial Intelligence ?!!!
- Simple Behaviors
- Complex Behaviors
- Conclusion
Introduction to State-Space

- **System Modeling**
- **Dynamical Systems**

\[
\dot{x} = f(x), \quad x(t_0) = x_0, \quad x(t) \in \mathbb{R}^m
\]

\[
x_{n+1} = f(x_n), \quad x_0 = x_0, \quad x_n \in \mathbb{R}^m
\]

- **Example: Logistic Map**

\[
x_{n+1} = ax_n(1 - x_n)
\]
Steady-State Behavior – I
Fixed Point

Logistic Map: \( a = 1.5 \), \( x(0) = 0.4 \), \( y(0) = 0.7 \)
Steady-State Behavior – II
Limit Cycle
Steady-State Behavior – II

... Limit Cycle

Logistic Map: $a=3.4 \quad x(0)=0.4 \quad y(0)=0.7$
Steady-State Behavior – IV
Quasi-periodic
Steady-State Behavior – V

Chaos

Logistic Map: $a=3.89, \ x(0)=0.4, \ y(0)=0.7$
Steady-State Behavior – V

... Chaos
Steady-State Behavior – V

... Chaos

Logistic Map: $a=3.89 \quad x(0)=0.7 \quad y(0)=0.7000001$
Definition of Chaos - I

Historical Perspective
- Laplace, 1776
- Poincare, 1903
- Birkhoff, 1920
- Kolmogorov, 1960
- Lorenz, 1963
- ...

Definition of Chaos - II

- None of the rest !!!

- Deterministic Randomness

- Unpredictable Causality
Chaotic Systems Properties - I

- Non-periodic Bounded Motion with Initial Condition Sensitivity
  - Butterfly Effect !!!
  - Restless Motion !!!
  - Heisenberg’s Uncertainty Principle
  - Naturally Unpredictable !!!
- Continuous, Broad-band, Noise-like Power Spectrum
Chaotic Systems Properties - II

- **Stretching and Folding**
  - Lyapunov Exponents
  - Growing Uncertainty

- **Information Creation and Destruction**
  - 2\(^{nd}\) Law of Thermodynamics Suspected
Chaotic Systems Properties - III

Bifurcation Diagram
Chaotic Systems Properties - III

... Bifurcation Diagram
Chaotic Systems Properties - III

... Bifurcation Diagram
Chaotic Systems Properties - IV

Strange Attractor
Chaotic Systems Properties - IV

... Strange Attractor
More Strange Attractors
More Strange Attractors
More Strange Attractors
Fractal Dimension

- What is Dimension?
- Why Euclidian?
- What is Fractal?
- What is Fractal Dimension
More Fractals

*Sierpinsky Triangle*
More Fractals

Fern
More Fractals

Even Completer
More Fractals

And Natural Ones